Thermal Boundary Layer on a Continuous Moving Plate with Freezing

F.-B. Cheung*
Pennsylvania State University, University Park, Pennsylvania

The growth of a solidified layer or "freeze coat" on the surface of a chilled continuous plate traveling steadily through a bath of warm liquid is investigated analytically. The behavior of the thermal boundary layer in the liquid flowfield that is induced by the motion of the plate is modeled along with the process of heat conduction in the solid phase to determine the location of the freezing front. Using the method of similarity, axial variations of the freeze-coat thickness and the coefficient of local convective heat transfer from the liquid to the solid phase are obtained as functions of various controlling parameters of the system. It is found that, while the shape of the freeze coat depends strongly on the local convective heat flux, the flow is, in turn, heavily influenced by the variation of the solid/liquid interface location. Because of this mutual interaction between the phase change process and the flow, the local convective heat-transfer coefficient at the freezing front is considerably larger than the corresponding value for the case of forced convection over a continuous moving plate without freezing. The effect of flow/freezing interaction is found to be quite pronounced, especially when the liquid Prandtl number is large and the freeze coat grows rapidly in the axial direction.

Nomenclature

C_{p_s} = specific heat of the freeze coat
C_{p_w} = specific heat of plate
f = reduced stream function, Eq. (13b)
h_x = local heat-transfer coefficient, Eq. (9)
k = thermal conductivity of the liquid
k_s = thermal conductivity of the freeze coat
k_w = thermal conductivity of the plate
Nu_x = local Nusselt number, Eq. (10)
Nu_{x_0} = local Nusselt number for the case without freezing, Eq. (11)
Pr = liquid Prandtl number, Eq. (20)
Re_x = local Reynolds number, Ux/ν
Ste = Stefan number, Eq. (20)
Ste_c = critical Stefan number
T = liquid temperature
T_f = freezing temperature of the liquid
T_0 = liquid temperature for the case without freezing
T_s = temperature of the freeze coat
T_w = temperature of the plate
T_{w_i} = plate inlet temperature
T_∞ = ambient liquid temperature
u = axial component of liquid velocity
U = plate velocity
v = vertical component of liquid velocity
x = axial coordinate measured from the inlet
y = vertical coordinate measured upward from the plate
α = thermal diffusivity of the liquid
α_s = thermal diffusivity of the freeze coat
α_w = thermal diffusivity of the plate
β = liquid superheat parameter, Eq. (20)
γ = thermal ratio between the freeze coat and the plate, Eq. (23)
δ = thickness of the freeze coat
δ^* = thickness of the freeze coat for the case without flow/freezing interaction
η = similarity variable, Eq. (13a)
θ = dimensionless liquid temperature, Eq. (13c)
λ = latent heat of fusion
ν = kinematic viscosity of the liquid
ρ = liquid density
ρ_s = freeze-coat density
σ = freeze-coating constant, Eq. (24)
σ^* = freeze-coating constant for the case without flow-freezing interaction, Eq. (30)
ϕ = dimensionless free-coat temperature, Eq. (13c)
Ψ = dimensionless plate temperature, Eq. (13c)

Subscripts

c = critical condition beyond which the effect of flow/freezing interaction is large
f = freezing condition
i = plate inlet condition
s = freeze coat
w = plate or wall
x = local axial quantity
∞ = ambient liquid condition
0 = absence of freezing

Superscripts

$(\ )^*$ = negligible flow/freezing interaction

Introduction

HEAT-TRANSFER problems involving a phase change due to solidification or melting are important in a variety of engineering applications such as the casting of metals, the drilling of high-ice-content soil, the storage of thermal energy, the ablation of heat shields for spacecraft, and the safety studies of nuclear reactors. One unique feature of this class of problems is that the freezing or melting front is usually unknown and is expressed implicitly in an equation for the conservation of thermal energy at the interface. Thus, the mathematical system governing the phase-change process is always nonlinear. Earlier studies of the phase-change problems, however, have been confined to the case in which the liquid phase is stagnant or the motion of the liquid is due entirely to the phase-change process itself. Under this condi-

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*Associate Professor, Department of Mechanical Engineering.

tion, the rate of freezing or melting is controlled by heat diffusion.[1-3]

Unlike the earlier work, most solid/liquid phase-change problems encountered in engineering practice do not occur in stagnant liquid media, but, instead, occur in a system in which the liquid phase is in motion relative to the solid phase.[4] In this case, the phase-change process can be strongly coupled to the liquid flowfield. While the location of the solid/liquid interface is heavily dependent upon the local convective heat transfer from the liquid, the flow in turn is influenced by the variation of the solid/liquid interface location. This mutual interaction represents a characteristic feature of phase change in fluid flow that is absent in the classical conduction-controlled phase-change problems. Because of the interaction between the flow and the moving interface, additional complications are introduced to the system besides the basic nonlinearity of the phase-change problem. As a result, the phenomena of solidification and melting in fluid flow can be substantially different from and more complicated than those in stagnant liquid media.

One important group of problems involving solid/liquid phase change in fluid flow that has been studied quite extensively is the one concerning freezing of a warm liquid suddenly flowing over the surface of a chilled stationary flat plate. For the case in which the plate is kept isothermal at a temperature below the freezing point of the liquid, the behavior of the frozen layer on the plate has been studied analytically by Libby and Chen,[5] Lapadula and Mueller,[6] Beaubouef and Chapman,[7] Savino and Siegel,[8] and Elmas.[9] For the case in which the plate is not isothermal but cooled below the freezing temperature of the liquid by a coolant flowing along the opposite side of the wall, the freezing process has been investigated experimentally by Savino and Siegel[10] and theoretically by Siegel and Savino[11,12] and Stephan.[13] Finally, the case of freezing of a flowing warm liquid on an initially chilled flat plate that is neither kept isothermal nor cooled by a coolant has been analyzed by Epstein[14] and El-Genk and Cronenberg.[15] In all of these studies, however, the convective heat flux from the warm liquid to the freezing front was not determined as a part of the solution by properly modeling the liquid flowfield. Rather, it was treated as an input quantity obtainable directly from the solution of the corresponding problem of forced convection without phase change by assuming the effect of flow/freezing interaction to be negligible. Under what conditions this assumption is valid have not been examined.

The problem of interest here is closely related to the one described above. It deals with the process of freezing of an otherwise quiescent liquid on a chilled, continuously moving plate. This solidification process, known as freeze coating,[16,17] finds application in the electrical and chemical industries, such as in the casting of a thin coating on metal plates or electricity cables. Depending on how a given product is being used in a particular application, the coating may have different functions. These include electrical insulation, high-temperature thermal insulation, rust resistance, corrosion resistance, wear resistance, and flame retardation. In actual practice, a cold metal plate is passed continuously through a bath of liquid. The moving plate is at a subcooled temperature below the freezing point of the liquid and, as it travels through the bath, freezing of the otherwise quiescent liquid takes place over its surface. The freeze-coating process differs from those studied in Refs. 5–15 in that the liquid flowfield is induced entirely by the motion of the plate and the thickness of the frozen layer or freeze coat varies spatially rather than with time. Because of the fact that the boundary layers that grow along moving objects are considerably different from those along stationary walls,[18-21] the results for the case of freezing of a flowing warm liquid on a stationary solid surface[5-15] cannot be used to optimize the freeze-coating conditions.

In spite of its practical importance, very few studies have been performed to investigate the fundamental aspects of the freeze-coating process. The papers by Seeniraj and Bose[16] and Cheung[17] appear to be the only work that deals with these type of problems. In the study of Seeniraj and Bose,[16] the moving object was assumed to remain at a constant uniform temperature during the freeze-coating process and the liquid was assumed to be saturated at its freezing temperature. The latter assumption eliminates the need to consider the liquid flowfield since the convective heat flux from the liquid to the solid boundary is identically zero everywhere. In the work of Cheung,[17] the requirements for constant object temperature and saturated liquid were relaxed. The process of freeze coating of a superheated liquid on a nonisothermal moving plate was investigated, taking into account the axial variation of the plate temperature and heat convection from the warm liquid to the moving plate. Based on the assumption that there was negligible interaction between the flow induced by the moving plate and the shape of the solid/liquid phase boundary, the solution of forced convection over a moving surface without phase change[19,20] was employed to determine the convective heat flux from the warm liquid to the freeze coat. This again eliminates the need to model the liquid flowfield. The work of Cheung,[17] although providing some useful results, does not address the important issue of the flow/freezing interaction. The conditions for which the forced convection solution of Refs. 19 and 20 for the case without phase change can be applied directly to the case with freezing have yet to be determined.

In this study, the process of freeze coating a superheated liquid on a thick, nonisothermal moving plate is investigated by accurately modeling the detailed liquid flowfield induced by the motion of the plate. The velocity and temperature profiles in the thermal boundary layer are determined simultaneously with the shape of the freeze coat. In so doing, the convective heat flux from the warm liquid to the solid boundary is obtained as a part of the solution. This is compared with the corresponding value for the case of forced convection over a moving plate without freezing.[19,20] The effect of the flow/freezing interaction that was ignored in Ref. 17 is estimated quantitatively for a wide variety of flow and heat-transfer conditions. Also estimated is the effect of the Prandtl number of the liquid, which was not addressed in Ref. 17.

Problem Formulation

The physical system to be considered in this study is depicted in Fig. 1. A continuous moving plate with a velocity U enters a large bath of liquid through a slit in an adiabatic bounding wall at $x = 0$. Before the immersion, the plate is at a subcooled temperature T_{w_i} below the freezing point T_f of the liquid. The ambient liquid is quiescent and is at a uniform temperature $T_\infty > T_f$. As the plate travels through the bath, a freeze coat forms on its surface. The density of the freeze coat ρ_s may be different from the density of the liquid ρ. The objectives are to determine the axial variation of the freeze-coat thickness $\delta(x)$ and the local convective heat flux $h_x(T_\infty - T_f)$

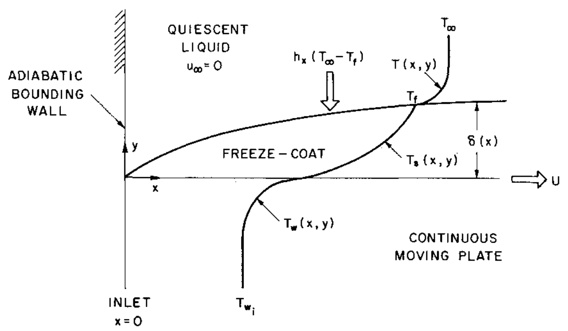

Fig. 1 Schematic of the physical configuration and the freeze coat along the continuous moving plate, indicating nomenclature.

from the liquid to the phase boundary at $y=\delta$. To formulate the problem, the frame of axes are chosen to be stationary with respect to the liquid bath. In addition, the following simplifying assumptions are employed:

(1) The freeze coating process is at a steady state. This, in turn, requires that both the plate velocity U and the ambient liquid temperature T_∞ remain constant in time.

(2) The process of freeze coating takes place at a constant temperature T_f. The solidification front is sharp and planar on the scale of the freeze-coat thickness; thermal equilibrium exists at the phase boundary.

(3) The thermophysical properties in a given phase are constant. However, there may be volumetric change upon freezing due to the difference in the liquid and the freeze-coat densities. This may induce a normal velocity at the freezing front.

(4) The liquid motion induced by the moving plate is steady and laminar and of the boundary-layer type. The latter condition has been observed in Ref. 20. The assumption of a laminar flow requires that the total immersion distance L of the plate be smaller than $\nu Re_c/U$, where ν is the kinematic viscosity of the liquid and Re_c the critical Reynolds number at which the transition occurs. This is a realistic assumption, since transition to turbulent flow is not the preferred freeze-coating operating condition as it may result in a sudden increase in the convective heat flux and thus a sharp reduction in the freeze-coat thickness. Therefore, in actual practice, it is desired to have the plate emerge from the liquid at a distance shorter than $\nu Re_c/U$.

(5) Over the entire immersion distance L, the freeze coat is very thin relative to the thickness of the plate itself. Thus, with respect to the freeze coat, the plate may be considered as a semi-infinite wall. This assumption allows the present results to be also applicable to the case of freezing coating on a moving cylinder with a diameter much larger than the thickness of the freeze coat.

(6) The effects of free convection and viscous dissipation are negligible. The latter is usually the case for laminar flows, whereas the former requires $Re^2 \gg Gr$, where Re and Gr are the Reynolds and Grashof numbers of the flow, respectively.

With the above assumptions, the equations governing the liquid velocity, liquid temperature, freeze-coat temperature, plate temperature, and axial variation of the freeze-coat thickness, can be written as follows:

Boundary-layer flow region $x \geqq 0$, $y \geqq \delta(x)$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$x=0:\ u=0,\ T=T_\infty > T_f \tag{4a}$$

$$y=\delta:\ u=U,\ v=\left(\frac{1-\rho_s}{\rho}\right)U\frac{\mathrm{d}\delta}{\mathrm{d}x},\ T=T_f \tag{4b}$$

$$y\to\infty:\ u=0,\ T=T_\infty \tag{4c}$$

Freeze-coat region $x \geqq 0, 0 \leqq y \leqq \delta(x)$

$$\rho_s C_{p_s} U\frac{\partial T_s}{\partial x} = k_s\frac{\partial^2 T_s}{\partial y^2} \tag{5}$$

$$x=0:\ \delta=0 \tag{6a}$$

$$y=0:\ T_s=T_w,\ k_s\frac{\partial T_s}{\partial y} = k_w\frac{\partial T_w}{\partial y} \tag{6b}$$

$$y=\delta:\ T_s=T_f,\ \rho_s U\lambda\frac{\mathrm{d}\delta}{\mathrm{d}x} = k_s\frac{\partial T_s}{\partial y} - k\frac{\partial T}{\partial y} \tag{6c}$$

Wall region $x \geqq 0$, $y \leqq 0$

$$\rho_w C_{p_w} U\frac{\partial T_w}{\partial x} = k_w\frac{\partial^2 T_w}{\partial y^2} \tag{7}$$

$$x=0:\ T_w=T_{w_i} < T_f \tag{8a}$$

$$y=0:\ T_w=T_s,\ k_w\frac{\partial T_w}{\partial y} = k_s\frac{\partial T_s}{\partial y} \tag{8b}$$

$$y\to-\infty:\ T_w=T_{w_i}\quad \text{or}\quad \frac{\partial T_w}{\partial y}=0 \tag{8c}$$

In the above formulation, the inlet plate temperature T_{w_i}, the ambient liquid temperature T_∞, and the plate velocity U are treated as constants. Consistent with the boundary-layer approximation, heat conduction in the axial direction is neglected in Eqs. (3), (5), and (7). With the plate moving at a constant speed, the left-hand sides of Eqs. (5) and (7) are actually the time rates of change of the enthalpies of the freeze coat and the wall, respectively, should we travel with the plate. The flow and the freeze coat are mutually coupled through the boundary conditions at the solid/liquid interface given by Eqs. (4b) and (6c). The condition for v at $y=\delta$ is due to the effect of volumetric change upon freezing. If the densities of the freeze coat and the liquid are the same, i.e., $\rho_s=\rho$, we have $v=0$ at $y=\delta$.

Inspection of Eqs. (1–8) indicates that the freeze-coat thickness is a function of the inlet plate temperature, ambient liquid temperature, plate velocity, and physical properties of the system including the liquid Prandtl number, density ratio between the freeze coat and the liquid, latent heat of fusion of the freeze coat, and heat capacity of the wall. The local coefficient h_x of the convective heat transfer from the liquid to the freeze coat can be calculated once the temperature distribution in the thermal boundary-layer region is determined. This is given by

$$h_x = \frac{k\left.\dfrac{\partial T}{\partial y}\right|_{y=\delta}}{T_\infty - T_f} \tag{9}$$

from which the local Nusselt number can be obtained as

$$Nu_x = \frac{h_x x}{k} = \frac{x\left.\dfrac{\partial T}{\partial y}\right|_{y=\delta}}{T_\infty - T_f} \tag{10}$$

Note that in Ref. 17 the value of Nu_x is assumed to be a given input quantity equal to the corresponding value Nu_{x_0} for the case of forced convection over a moving plate without freezing.[19,20] As such, there is no need to consider the flowfield. In this study, the correct value of Nu_x will be determined directly from Eqs. (1–10). This will be compared with the corresponding value Nu_{x_0} to assess the effect of flow/freezing interaction. The value of Nu_{x_0} is given by

$$Nu_{x_0} = \frac{x\left.\dfrac{\partial T_0}{\partial y}\right|_{y=0}}{T_\infty - T_f} \tag{11}$$

where $(\partial T_0/\partial y)\ y=0$ can be obtained directly from Eqs. (1–4) with T being replaced by T_0 and Eq. (4c) being replaced by the following condition:

$$y=0:\ u=U,\ v=0,\ T_0=T_f \tag{12}$$

Note that in the absence of freezing, the solid/liquid interface is always located at $y=0$ and there is no need to consider Eqs. (5–8). The values of Nu_{x_0} have been obtained for larger Prandtl numbers in Ref. 19 and for moderate-to-large Prandtl numbers in Ref. 20.

Analysis

Equations (1–8) can be transformed, using the method of similarity, into a system of ordinary differential equations that can be readily solved by a standard numerical technique. To do this, the following similarity transformation is invoked:

$$\eta = \frac{y}{\delta}, \; \delta = \sigma\left(\frac{\nu x}{U}\right)^{\frac{1}{2}} \tag{13a}$$

$$u = Uf'(\eta), \; v = \frac{1}{2}\sigma\left(\frac{\nu U}{x}\right)^{\frac{1}{2}}(\eta f' - f) \tag{13b}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \; \phi(\eta) = \frac{T_s - T_{w_i}}{T_f - T_{w_i}}, \; \Psi(\eta) = \frac{T_w - T_{w_i}}{T_f - T_{w_i}} \tag{13c}$$

where f is the reduced stream function, η the independent similarity variable, and σ the freeze-coating constant. With the above transformation, Eqs. (1) and (6a) are satisfied automatically. Since we have assumed the freeze coat to be very thin and treated the plate as a semi-infinite wall, the plate can be viewed as an infinite heat sink. As a result, the freeze coat grows monotonically in the axial direction according to Eq. (13a). If, however, the freeze coat is not thin compared to the plate, then the plate is no longer equivalent to an infinite heat sink and its bulk temperature will increase in the axial direction. As the plate warms up after traveling a certain distance from the inlet, remelting of the freeze coat will occur. This interesting behavior of growth and decay has been studied in Ref. 17.

In the similarity coordinate, the boundary-layer region is given by $\eta \geqq 1$, the freeze-coat region by $0 \leqq \eta \leqq 1$, and the wall region by $\eta \leqq 0$. Substituting Eq. (13) into Eqs. (2–8), we obtain

$$f''' + \frac{\sigma^2}{2} ff'' = 0 \quad \text{for } \eta \geqq 0 \tag{14}$$

$$\theta'' + \frac{\sigma^2}{2} Prf\theta' = 0 \quad \text{for } \eta \geqq 1 \tag{15}$$

$$\phi'' + \frac{\sigma^2}{2}\left(\frac{\alpha}{\alpha_s}\right) Pr\eta\phi' = 0 \quad \text{for } 0 \leqq \eta \leqq 1 \tag{16}$$

$$\Psi'' + \frac{\sigma^2}{2}\left(\frac{\alpha}{\alpha_w}\right) Pr\eta\Psi' = 0 \quad \text{for } \eta \leqq 0 \tag{17}$$

with the boundary conditions

$$f'(1) = \theta(1) = \phi(1) = 1 \tag{18a}$$

$$f'(\infty) = \theta(\infty) = \Psi(-\infty) = 0 \tag{18b}$$

$$f(1) = \rho_s/\rho, \; \phi(0) = \Psi(0)$$

$$\phi'(0) = (k_w/k_s)\Psi'(0) \tag{18c}$$

where the primes denote total derivatives with respect to η. The freeze-coating constant is given by

$$\sigma = \left\{2\left(\frac{\alpha}{\alpha_s}\right)^{-1} Pr^{-1}Ste[\phi'(1) + \beta\theta'(1)]\right\}^{\frac{1}{2}} \tag{19}$$

where Pr, Ste, and β are the liquid Prandtl number, Stefan number, and liquid superheat parameter defined, respectively, by

$$Pr = \frac{\nu}{\alpha}, \; Ste = \frac{C_{p_s}(T_f - T_{w_i})}{\lambda}, \; \beta = \frac{k(T_\infty - T_f)}{k_s(T_f - T_{w_i})} \tag{20}$$

In terms of the unknown quantity σ, Eqs. (16) and (17) may be integrated analytically using the appropriate boundary conditions given by Eq. (18) to get

$$\phi = \frac{\operatorname{erf}\left[\frac{\sigma}{2}\left(\frac{\alpha}{\alpha_s}Pr\right)^{\frac{1}{2}}\eta\right] + \gamma}{\operatorname{erf}\left[\frac{\sigma}{2}\left(\frac{\alpha}{\alpha_s}Pr\right)^{\frac{1}{2}}\right] + \gamma} \quad \text{for } 0 \leqq \eta \leqq 1 \tag{21}$$

$$\Psi = \frac{\gamma\left\{1 - \operatorname{erf}\left[-\frac{\sigma}{2}\left(\frac{\alpha}{\alpha_w}Pr\right)^{\frac{1}{2}}\eta\right]\right\}}{\operatorname{erf}\left[\frac{\sigma}{2}\left(\frac{\alpha}{\alpha_s}Pr\right)^{\frac{1}{2}}\right] + \gamma} \quad \text{for } \eta \leqq 0 \tag{22}$$

where γ is the thermal ratio between the freeze coat and the plate defined by

$$\gamma = (k_s\rho_s C_{p_s}/k_w\rho_w C_{p_w})^{\frac{1}{2}} \tag{23}$$

Substituting Eq. (21) into Eq. (19), we obtain:

$$\sigma = \left\{2\left(\frac{\alpha}{\alpha_s}\right)^{-1} Pr^{-1}Ste \times \left[\beta\theta'(1) + \frac{\frac{\sigma}{\pi}\left(\frac{\alpha}{\alpha_s}Pr\right)^{\frac{1}{2}}\exp\left(-\frac{\sigma^2}{4}\frac{\alpha}{\alpha_s}Pr\right)}{\operatorname{erf}\left[\frac{\sigma}{2}\left(\frac{\alpha}{\alpha_s}Pr\right)^{\frac{1}{2}}\right] + \gamma}\right]\right\}^{\frac{1}{2}} \tag{24}$$

The exact value of $\theta'(1)$ has to be determined from Eqs. (14), (15), and (18). To facilitate the procedure of numerical computation, it is desired to express $\theta'(1)$ in terms of σ. Rearranging Eq. (24), we have

$$\theta'(1) = \frac{1}{\beta}\left\{\frac{1}{2}\sigma^2\left(\frac{\alpha}{\alpha_s}\right)PrSte^{-1} - \frac{\frac{\sigma}{\sqrt{\pi}}\left(\frac{\alpha}{\alpha_s}Pr\right)^{\frac{1}{2}}\exp\left(-\frac{\sigma^2}{4}\frac{\alpha}{\alpha_s}Pr\right)}{\operatorname{erf}\left[\frac{\sigma}{2}\left(\frac{\alpha}{\alpha_s}Pr\right)^{\frac{1}{2}}\right] + \gamma}\right\} \tag{25}$$

Once we know the values of σ and $\theta'(1)$, the local Nusselt number at the solid/liquid interface, $\eta = 1$, can be determined from Eq. (10). This is

$$Nu_x = \frac{-\theta'(1)}{\sigma} Re_x^{\frac{1}{2}} \tag{26}$$

where $Re_x = Ux/\nu$ is the local Reynolds number of the flow. Note from Eqs. (18a) and (18b) that $\theta'(1)$ is a negative quantity and thus $Nu_x > 0$.

Inspection of Eqs. (14), (15), (18), (24), and (26) indicate that σ and Nu_x are functions of six independent controlling parameters. These are the liquid Prandtl number Pr, Stefan number Ste, liquid superheat parameter β, thermal ratio γ, diffusivity ratio α/α_s, and density ratio ρ_s/ρ. Note from Eq. (14) that the product $\sigma Pr^{\frac{1}{2}}$ cannot be selected as an appropriate parameter to simplify the dependence of σ on Pr. This is due to the fact that the second term on the left-hand side of Eq. (14) involves only σ^2 but not the product $\sigma^2 Pr$. For given values of Pr, Ste, β, γ, α/α_s, and ρ_s/ρ, Eqs. (14) and (15) may be integrated numerically along with Eqs. (18) using the Runge-Kutta method. The correct value of σ is chosen such that the value of $\theta'(1)$ so determined is identical to the one given by Eq. (25). The corresponding value of $Nu_x/Re_x^{\frac{1}{2}}$ can then be obtained from Eq. (26). This is to be compared with the value of $Nu_{x_0}/Re_x^{\frac{1}{2}}$ for the case without freezing. According to the results reported in Refs. 19 and 20, the quantity $Nu_{x_0}/Re_x^{\frac{1}{2}}$ can be represented by

$$Nu_{x_0}/Re_x^{\frac{1}{2}} = CPr^{\frac{1}{2}} \tag{27}$$

where the coefficient C is a weak function of the Prandtl number having the values of 0.4174, 0.4438, 0.5314, and 0.5545, respectively, for the cases of $Pr = 0.7$, 1.0, 10, and 100. From Eqs. (26) and (27), we have

$$\frac{Nu_x}{Nu_{x_0}} = -\frac{\theta'(1)/\sigma}{CPr^{\frac{1}{2}}} \tag{28}$$

If the effect of the flow/freezing interaction is indeed negligible, we should have $Nu_x = Nu_{x_0}$. Thus, any deviations of (Nu_x/Nu_{x_0}) from unity can be used quantitatively to estimate how strongly the convective heat flux from the flow to the phase boundary is coupled to the freezing process. Note that should we ignore the effect of flow/freezing interaction, we may set

$$Nu_x = Nu_{x_0} \quad \text{or} \quad \theta'(1) = -CPr^{\frac{1}{2}}\sigma \tag{29}$$

Substituting the above expression into Eq. (24), we obtain, after some manipulations

$$\sigma^* = 2\left(\frac{\alpha}{\alpha_s}\right)^{-1} Pr^{-\frac{1}{2}} Ste \times \left[-C\beta + \frac{\frac{1}{\sqrt{\pi}}\left(\frac{\alpha}{\alpha_s}\right)^{\frac{1}{2}} \exp\left(-\frac{\sigma^{*2}}{4}\frac{\alpha}{\alpha_s}Pr\right)}{\operatorname{erf}\left[\frac{\sigma^*}{2}\left(\frac{\alpha}{\alpha_s}Pr\right)^{\frac{1}{2}}\right] + \gamma} \right] \tag{30}$$

where σ^* represents the value of the freeze-coating constant for the case without the flow-freezing interaction. From Eq. (13a), the corresponding freeze-coat thickness is simply given by

$$\delta^* = \sigma^*(\nu x/U)^{\frac{1}{2}} \tag{31}$$

For a given set of Pr, Ste, β, γ, and α/α_s, the value of δ^* can be determined directly from Eqs. (30) and (31) without having to consider the flow behavior or to integrate Eqs. (14) and (15). Should there be no flow/freezing interaction, the mathematical solution procedure would be greatly simplified. Unfortunately, as discussed in the next section, the assumption of $Nu_x = Nu_{x_0}$ is valid only under certain flow and heat-transfer conditions. In general, Nu_x is considerably higher than Nu_{x_0} and the value of σ has to be obtained from Eq. (24) by solving simultaneously for the liquid velocity and temperature profiles in the thermal boundary layer on the moving plate.

Results and Discussion

As mentioned above, the primary objective of this study is to determine qualitatively and quantitatively the effect of flow/freezing interaction. This can be achieved by comparing the values of the freeze-coating constant σ and the local Nusselt number Nu_x with the corresponding values of σ^* and Nu_{x_0} obtained from the solution of forced convection flow over a moving plate without freezing. In this section, we shall first examine how strongly the flow depends on the freeze-coating process and then determine how the flow affects the shape of the freeze coat. It is evident from Eqs. (14), (15), (18), and (26) that the value of $Nu_x/Re_x^{\frac{1}{2}}$ is completely determined once σ, Pr, and ρ_s/ρ are given. This is true even though σ itself is a function of Ste, Pr, α/α_s, ρ_s/ρ, γ, and β. Physically, σ is directly proportional to the rate of freezing. Hence, it is convenient for us to examine how the local Nusselt

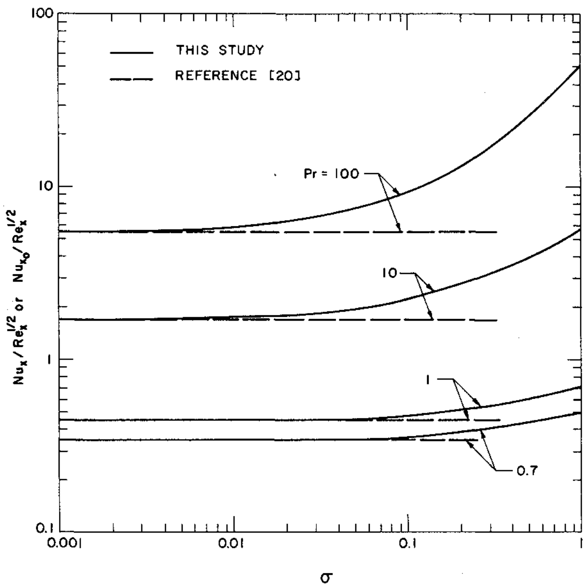

Fig. 2 Variation of $Nu_x/Re_x^{\frac{1}{2}}$ with σ for different values of Pr ($\rho_s/\rho = 1$): comparison with the corresponding values of $Nu_{x_0}/Re_x^{\frac{1}{2}}$ as $\sigma \to 0$.

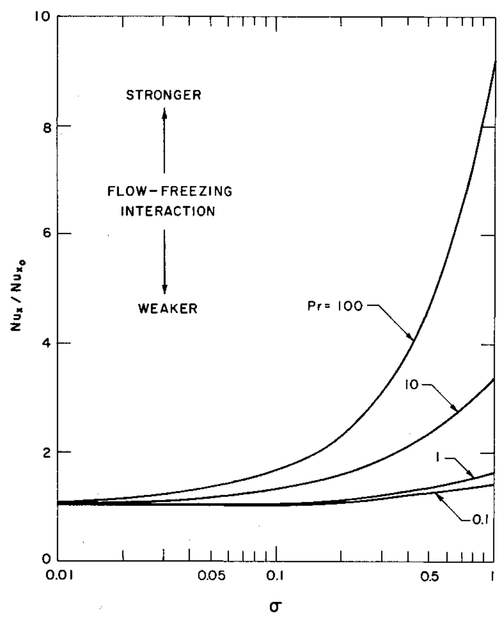

Fig. 3 Effect of flow/freezing interaction on the local Nusselt number: variation of Nu_x/Nu_{x_0} with σ for different values of Pr ($\rho_s/\rho = 1$).

number varies with the freeze-coating constant by treating σ as an independent parameter. In doing so, the effect of freezing on the convective heat flux can be adequately assessed.

Figure 2 shows the variation of $Nu_x/Re_x^{\frac{1}{2}}$ with σ (solid curves) for four different values of Pr with ρ_x/ρ fixed at unity. Also shown in the figure for comparison are the corresponding values of $Nu_{x_0}/Re_x^{\frac{1}{2}}$ reported in Ref. 20 (dashed lines). For a given Prandtl number, Nu_x approaches asymptotically to Nu_{x_0} as σ approaches zero. This result is quite expected since there is virtually no effect of freezing on the flow if σ is very small. Under this circumstance, the local convective heat flux becomes the same as the one for the case without phase change.[20] As the value of σ is increased, however, Nu_x gets larger than Nu_{x_0}. This indicates that the presence of freezing would enhance the local convective heat-transfer rate. For a given value of σ, a larger difference between Nu_x and Nu_{x_0} is obtained at a higher Prandtl number, implying that the effect of flow/freezing interaction is more pronounced as the value of Pr is increased.

The dependence of the local convective heat-transfer coefficient on the freeze-coating process is further illustrated in Fig. 3, where the ratio Nu_x/Nu_{x_0} is plotted vs σ for four different values of Pr. Again, ρ_s/ρ is fixed at unity. Apparently, the assumption of $Nu_x = Nu_{x_0}$ is valid only when both σ and Pr are sufficiently small. At higher values of σ and Pr, the effect of flow/freezing interaction is strong and Nu_x/Nu_{x_0} becomes much larger than unity. In fact, at $\sigma = 1$ and $Pr = 100$, Nu_x is almost an order of magnitude higher than Nu_{x_0}. Thus, for larger values of Pr and σ, the convective heat flux obtained from the solution of forced convection over a moving plate without phase change[19,20] could be considerably smaller than the actual heat flux convected from the liquid to the freeze coat. Evidently, the assumption of $Nu_x/Nu_{x_0} = 1$ could be grossly in error, especially when the liquid Prandtl number is large and the freeze coat grows rapidly in the axial direction.

The effect of ρ_s/ρ on the local convective heat transfer from the liquid to the freeze coat has also been studied. For a given set of Pr and σ, it is found that the local Nusselt number Nu_x decreases slightly with decreasing value of ρ_s/ρ. This implies that the local convective heat flux would be somewhat lower should the liquid expand upon freezing. In most cases, however, the value of ρ_s/ρ is very close to unity. This is true even for substances like water ($\rho_s/\rho = 0.92$). Hence, the effect of volumetric change can usually be ignored and there is no need to consider this parameter further. Using a similar argument, the parameter α/α_s can also be eliminated for simplicity. In what follows, we shall focus on the effects of Pr, Ste, γ, and β by setting $\rho_s/\rho = \alpha/\alpha_s = 1$.

The calculated variations of the freeze-coating constant σ with the four independent parameters, Pr, Ste, γ, and β, are depicted in Figs. 4 and 5. Since most freeze-coating materials have relatively large Prandtl numbers, it is practical to focus on the high Prandtl number behavior. For this purpose, the values of 10 and 100 are chosen for Pr. In Fig. 4, the liquid superheat parameter is fixed at $\beta = 0.1$. As expected, σ is a monotonically increasing function of Ste. Thus, a decrease in either the latent heat of fusion or the wall temperature will speed up the growth of the freeze coat in the axial direction. On the other hand, σ becomes considerably smaller as the value of Pr is increased. This result is consistent with those presented in Figs. 2 and 3. As Pr gets larger, there is a marked increase in the local convective heat flux, thus slowing down the growth of the freeze coat. For a given set of Ste and Pr, σ tends to decrease with increasing value of γ. Physically, this is expected, since an increase in the value of γ corresponds to a decrease in the wall cooling capability. Note that σ is sensitive to the change in the value of γ only for small-to-moderate Stefan numbers. For $Ste \geqq 10$, σ becomes almost independent of γ.

The effects of β, Ste, and Pr on the freeze-coating constant are shown in Fig. 5, with the freeze coat-to-wall thermal ratio maintained constant at $\gamma = 0.1$. Evidently, for a given value of β, the functional dependence of σ on Ste and Pr is very similar to the one depicted in Fig. 4. For a given set of Ste and Pr, on the other hand, σ tends to decrease with increasing value of β. Physically, the liquid superheat has a tendency to hinder the growth of the freeze coat. Hence, a larger value of β will result in a smaller value of σ. Note that σ is sensitive to the change in the value of β only for large-to-moderate Stefan numbers. For $Ste \leqq 0.01$, σ becomes almost independent of β. This behavior is just opposite to the one associated with γ (see Fig. 4). A possible explanation is that at small Stefan numbers, the latent heat effect dominates the sensible heat effect. Thus, the freeze-coating process is strongly dependent on the wall cooling condition, while only weakly dependent on the liquid superheat condition. On the contrary, at large Stefan numbers, the sensible heat effect dominates the

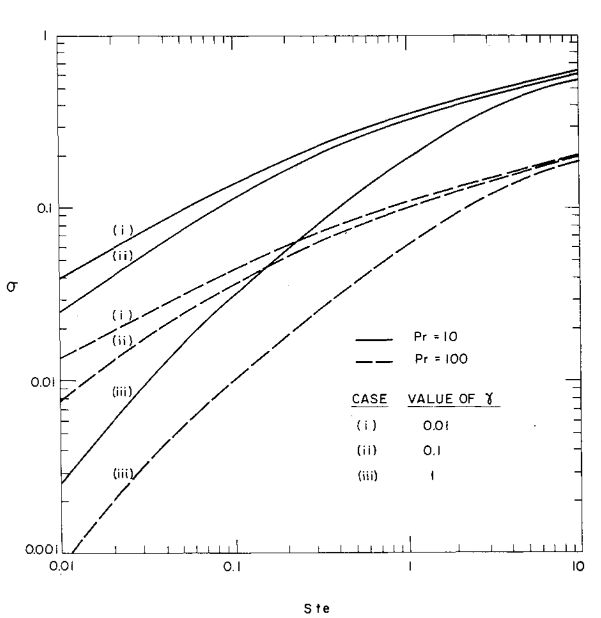

Fig. 4 Effects of the Stefan number, liquid Prandtl number, and thermal ratio on the freeze-coating constant ($\beta = 0.1$).

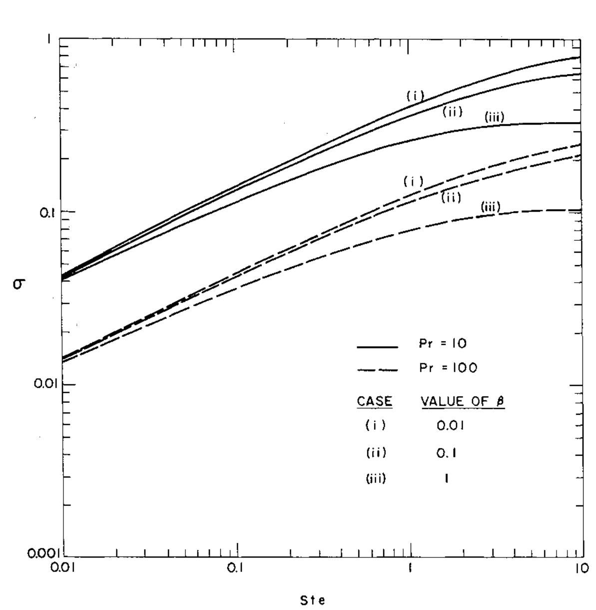

Fig. 5 Effects of the Stefan number, liquid Prandtl number, and liquid superheat parameter on the freeze-coating constant ($\gamma = 0.1$).

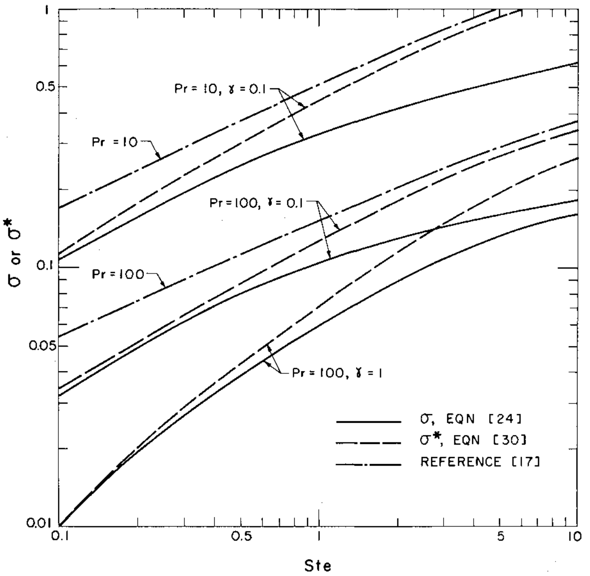

Fig. 6 Effect of flow/freezing interaction on the freeze-coating constant: variations of σ and σ^* with *Ste*, *Pr*, and γ ($\beta = 0.1$).

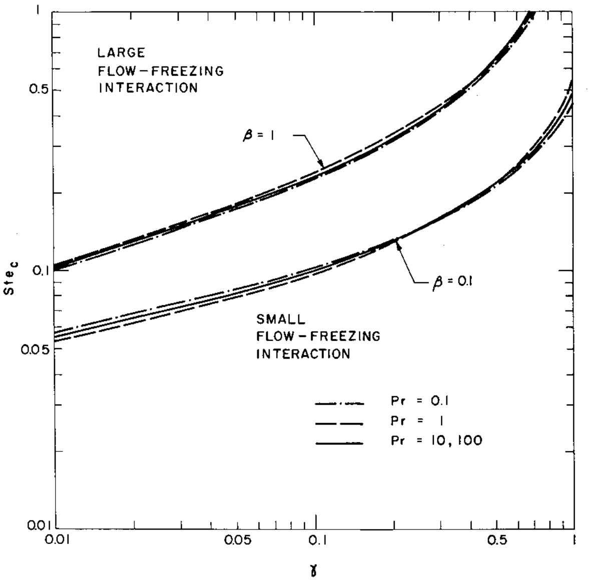

Fig. 7 Variation of the critical Stefan number with the thermal ratio, liquid superheat parameter, and liquid Prandtl number.

latent heat effect. Therefore, the freeze-coating process becomes more sensitive to the liquid superheat condition.

The values of σ and σ^*, calculated respectively from Eqs. (24) and (30), are presented in Fig. 6 for the case of $\beta = 0.1$. Also presented in the figure are the corresponding first-order solutions reported in Ref. 17. These solutions were obtained for the case of an isothermal plate (i.e., $\gamma = 0$) by assuming $Nu_x = Nu_{x_0}$. As mentioned above, the presence of freezing could substantially enhance the coefficient of local convective heat transfer from the liquid to the freeze coat. Therefore, it is of fundamental interest and practical importance to estimate the error involved in the calculated freeze-coating constant should the effect of flow/freezing interaction be ignored. As can be seen in Fig. 6, the behavior of σ^* is qualitatively similar to the behavior of σ. However, the value of σ^* is always larger than the corresponding value of σ. In fact, the difference between σ^* and σ becomes too large to be tolerated at high Stefan numbers. The same conclusion can be drawn when comparing the value of σ with the corresponding value predicted in Ref. 17. Evidently, the assumption of $Nu_x = Nu_{x_0}$ could significantly overpredict the growth of the freeze coat in the axial direction. This clearly demonstrates that, in general, the solution of forced convection over a moving plate without freezing[19,20] cannot be correctly employed to calculate the local convective heat flux and the freeze-coat thickness.

To determine under what conditions the flow/freezing interaction is strong and under what conditions its effect can be ignored, it is convenient to arbitrarily set

$$(\sigma^* - \sigma)/\sigma \leq 10\% \tag{32}$$

as the criterion. The critical Stefan number Ste_c, defined as the value of *Ste* above which the condition given by Eq. (32) cannot be satisfied, may then be obtained as a function of *Pr*, γ, and β. Results for the case of $\rho_s/\rho = \alpha_s/\alpha = 1$ are shown in Fig. 7. Although both the local Nusselt number Nu_x and the freeze-coating constant σ depend strongly on the Prandtl number, the critical Stefan number is found to be quite insensitive to the value of *Pr*. This surprising result can be explained as follows. From Figs. 2 and 3, it is obvious that the difference between Nu_x and Nu_{x_0} depends on both the Prandtl number and the growth rate of the freeze coat. The difference between Nu_x and Nu_{x_0} actually diminishes as σ approaches to zero, whereas it becomes larger as the Prandtl number is increased. For a given set of *Ste*, γ, and β, however, an increase in the value of *Pr* is always accompanied by a decrease in the value of σ, as can be seen in Figs. 4 and 5. Thus, the amount of increase in the difference between Nu_x and Nu_{x_0} due to increasing value of *Pr* is completely offset by the amount of decrease in the difference between Nu_x and Nu_{x_0} due to decreasing value of σ. As a result, the critical condition represented by $Ste_c(\gamma, \beta)$ is almost independent of *Pr*. It should be noted that, in most freeze-coating operations, the value of β is usually smaller than 0.1, whereas the value of γ is smaller than 0.05. It follows from Fig. 7 that the effect of flow/freezing interaction can be ignored only if the Stefan number is less than 0.08. In general, the requirement of $Ste < 0.08$ is not met and, therefore, the solution of forced convection over a moving plate without freezing[19,20] cannot be employed to determine the local convective heat-transfer rate. The behavior of the freeze coat has to be determined simultaneously with the behavior of the thermal boundary layer in the liquid phase.

Summary and Conclusions

We have investigated analytically the problem of freeze coating on a continuous moving plate. More specifically, we have identified the various dimensionless parameters that control the shape of the freeze coat and the behavior of the thermal boundary layer. Also, we have demonstrated the important effect of flow/freezing interaction on the coefficient of local convective heat transfer from the liquid to the solid phase and on the growth of the freeze coat. In general, the freeze-coating constant σ, which measures the rate of growth of the freeze coat in the axial direction, is a strong function of the liquid Prandtl number *Pr*, the Stefan number *Ste*, the liquid superheat parameter γ, and the thermal ratio β between the freeze coat and the wall. Increasing the value of *Ste* results in a larger value of σ, whereas increasing the value of either *Pr*, γ, or β results in a smaller value of σ. At small Stefan numbers, however, σ is more sensitive to the variation in the value of γ than β. On the contrary, at high Stefan number σ is more sensitive to the variation in the value of β than γ.

The effect of flow/freezing interaction, as measured by the differences between the actual value of the local Nusselt number Nu_x and the corresponding value Nu_{x_0} calculated from the solution of forced convection over a moving plate without freezing,[19,20] is negligible only when the rate of growth of the freeze coat in the axial direction is very small, i.e., $\sigma \ll 1$. For moderate values of σ, the effect of the flow/freezing interaction can be quite significant, especially at high Prandtl numbers. Under such circumstances, the value of Nu_x can be substantially larger than the corresponding value of Nu_{x_0}. Thus, the solution of forced convection over a moving plate without freezing[19,20] cannot be employed to calculate the convective heat flux at the freezing front, as it would greatly overpredict the local freezing-coat thickness. The shape of the freeze coat has to be determined by simultaneously solving the equations governing the temperature field in the solid phase and the equations governing the behavior of the thermal boundary layer in the liquid phase.

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References

[1] Bankoff, S.G., "Heat Conduction or Diffusion with Change of Phase," *Advances in Chemical Engineering*, Vol. 5, 1964, pp. 75–150.

[2] Muehlbauer, J.C. and Sunderland, J.E., "Heat Conduction with Freezing or Melting," *Applied Mechanics Reviews*, Vol. 18, 1965, pp. 951–959.

[3] Ockendon, J.R. and Hodgkins, W.R., *Moving Boundary Problems in Heat Flow and Diffusion*: *Proceedings*, Oxford Press, London, 1975.

[4] Cheung, F.B. and Epstein, M., "Solidification and Melting in Fluid Flow," *Advances in Transport Processes*, Vol. 3, 1984, pp. 35–117.

[5] Libby, P.A. and Chen, S., "The Growth of a Deposited Layer on a Cold Surface," *International Journal of Heat and Mass Transfer*, Vol. 8, 1965, pp. 395–402.

[6] Lapadula, C. and Mueller, W.K., "Heat Conduction with Solidification and a Convective Boundary Condition at the Freezing Front," *International Journal of Heat and Mass Transfer*, Vol. 9, 1966, pp. 702–705.

[7] Beaubouef, R.T. and Chapman, A.J., "Freezing of Liquids in Forced Flow," *International Journal of Heat and Mass Transfer*, Vol. 10, 1967, pp. 1581–1588.

[8] Savino, J.M. and Siegel, R., "An Analytical Solution for Solidification of a Moving Warm Liquid onto an Isothermal Cold Wall," *International Journal of Heat and Mass Transfer*, Vol. 12, 1969, pp. 803–809.

[9] Elmas, M., "On the Solidification of a Warm Liquid over a Cold Wall," *International Journal of Heat and Mass Transfer*, Vol. 13, 1970, pp. 1060–1062.

[10] Savino, J.M. and Siegel, R., "Experimental Study of the Transient Solidification of a Warm Liquid Flowing over a Chilled Flat Plate," NASA TND-4015, 1967.

[11] Siegel, R. and Savino, J.M., "An Analysis of the Transient Solidification of Flowing Warm Liquid on a Convectively Cooled Wall," *Proceedings of Third International Heat Transfer Conference*, Vol. 4, 1966, pp. 141–151.

[12] Siegel, R., and Savino, J.M., "Transient Solidification of a Flowing Liquid on a Cold Plate Including Heat Capacities of Frozen Layer and Plate," NASA TND-4353, 1968 (also ASME Paper 67-WA/HT-34, 1967).

[13] Stephan, K., "Influence of Heat Transfer on Melting and Solidification in Forced Flow," *International Journal of Heat and Mass Transfer*, Vol. 12, 1969, pp. 199–214.

[14] Epstein, M., "The Growth and Decay of a Frozen Layer in Forced Flow," *International Journal of Heat and Mass Transfer*, Vol. 19, 1976, pp. 1281–1288.

[15] El-Genk, M.S. and Cronenberg, A.W., "Stefan-Like Problems in Finite Geometry," *AIChE Symposium Series*, No. 189, Vol. 75, 1979, pp. 69–80.

[16] Seeniraj, R.V. and Bose, T.K., "Freeze-Coating on Continuous Moving Sheet and on an Axially Moving Cylinder," *Wärme- und Stoffübertragung*, Vol. 15, 1981, pp. 239–243.

[17] Cheung, F.B., "Analysis of Freeze Coating on a Nonisothermal Moving Plate by a Perturbation Method," *Journal of Heat Transfer*, Vol. 107, 1985, pp. 549–556.

[18] Sakiadis, B.C., "Boundary-Layer Behavior on Continuous Solid Surfaces: II. The Boundary Layer on a Continuous Flat Surface," *AIChE Journal*, Vol. 7, 1961, pp. 221–225.

[19] Erickson, L.E., Cha, L.C., and Fan, L.T., "The Cooling of a Moving Continuous Flat Sheet," *AIChE Chemical Engineering Progress Symposium Series*: *Heat Transfer*, Vol. 62, 1966, pp. 157–165.

[20] Tsou, F.K., Sparrow, E.M., and Goldstein, R.J., "Flow and Heat Transfer in the Boundary Layer on a Continuous Moving Surface," *International Journal of Heat and Mass Transfer*, Vol. 10, 1967, pp. 219–235.

[21] Moutsoglou, A. and Bhattacharya, A.K., "Laminar and Turbulent Boundary Layers on Moving, Nonisothermal Continuous Flat Surfaces," *Journal of Heat Transfer*, Vol. 104, 1982, pp. 707–714.